COT 3100 In-class Exercise 7

Name: USF ID:

Problem 1: Prove the statement by contraposition:

**For all integers *n*, if is divisible by 5, then is divisible by 5.**

Proof by contraposition:

Suppose n is any integer that is not divisible by 5. [we must show that is not divisible by 5.]

By quotient-remainder theorem, n can be written in one of the forms

For some integer. In fact, since n is not divisible by 5, n must have one of the forms

Case 1 (): since , by substitution. Thus, is not divisible by 5 by definition of divisibility.

Case 2 (): since , 4 by substitution. Thus, is not divisible by 5 by definition of divisibility.

Case 3 (): since, = by substitution. Thus, is not divisible by 5 by definition of divisibility.

Case 4 (): since , by substitution. Thus, is not divisible by 5 by definition of divisibility.

Conclusion: cases 1, 2, 3 and 4 show that is not divisible by 5 [as was to be shown].

Problem 2: Prove the statement by contradiction.

**is irrational.**

Proof by contradiction:

Suppose is rational. By definition of rational,. We also assume and have no common factor. Squaring both sides of and multiplying both sides by gives

(1)

Thus, is divisible by 5 by definition of divisibility. So, by the conclusion of problem 1, is divisible by 5. By definitional of divisibility, then, for some integer, and so,

(2)

Substituting equation (2) into equation (1), gives , and by dividing both sides by 5, yields

Hence, is divisible by 5 by definition of divisibility. So, by the conclusion of problem 1, is divisible by 5.

Consequently, both and have common factor 5, which contradict the assumption that and have no common factor. Thus, the supposition is false, and so, is irrational.